University Defence Research Collaboration in Signal Processing

LSSC Consortium White Paper

Polynomial Eigenvalue Decomposition (PEVD) Algorithms for Broadband Sensor Array Processing

Introduction

Sensor array processing is concerned with gathering and combining data from a collection of sensors to perform estimation of signal and environmental parameters through spatial and temporal processing. Array geometry allows the estimation of parameters that can be used to inform on the nature of signal transmissions and communications (e.g. Direction of Arrival, Source Separation) and can provide the basis for further processing, e.g. adaptive beamforming to strengthen signal transmission/reception or to attenuate unwanted signals or clutter interference through null-steering. Sensor arrays form the basis of an increasingly wide variety of applications including Multiple-Input Multiple-Output (MIMO) Communications, Radar and Sonar systems. Incoming data to the sensor array are formed into a measurement, analysis and steering vectors and processed using linear algebra techniques.

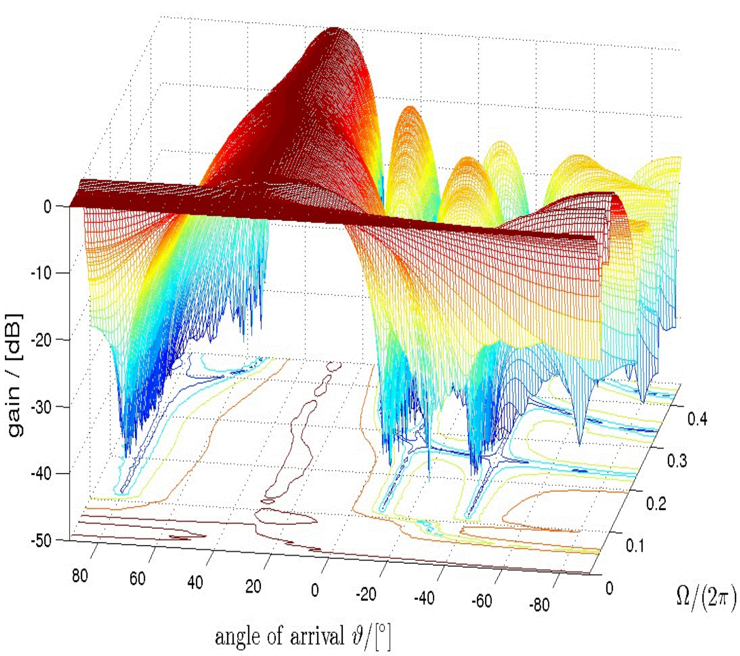

Figure 1: gain response of a broadband beamformer designed via polynomial matrix formulations to receive a signal of interest from 30 degrees while suppressing three broadband interferers (-40, -10 and 80 degrees).

For processing sensor array data, matrix decomposition techniques such as the eigenvalue decomposition (EVD) have been proven optimal for many different narrowband problems. An eigenvalue decomposition allows a Hermitian matrix to be factorised, e.g. to reveal subspace decompositions that are useful for data compression and direction or arrival applications. In the narrowband case, the propagation delay experienced by signals when travelling across the sensor array can be modelled sufficiently by a phase shift. In recent years research has sought to extend such techniques toward broadband sensor array processing. Our research has investigated the use of polynomial matrix techniques to tackle this objective, as they provide a data representation of greater complexity allowing full time delays to be modeled rather than just a phase shifts. Traditional broadband techniques would typically employ narrowband techniques to multiple individual frequency bins spanning the range of interest, thus leading to a lack of coherence/discontinuous estimates.

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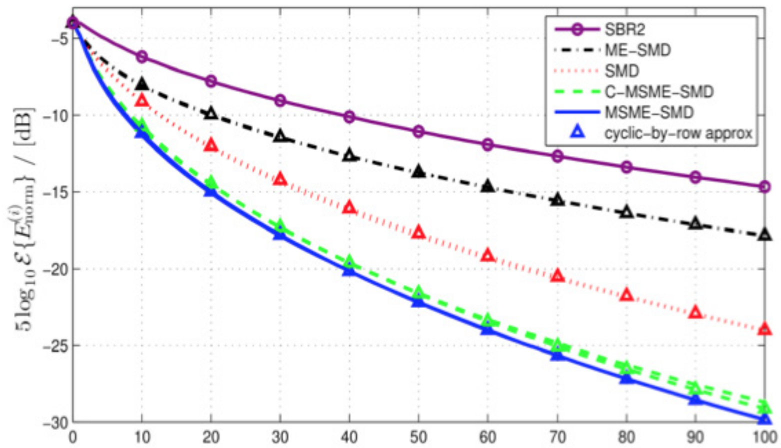
Method

The polynomial matrix EVD (PEVD) is an extension of the EVD for polynomial matrices. The PEVD facilitates the factorization of a para-Hermitian polynomial matrix into a product consisting of a diagonal polynomial matrix that is pre- and post-multiplied by paraunitary matrices. A number of different iterative PEVD algorithms have been in development, including Sequential-Matrix-Decomposition (SMD) [1], and Multiple-Shift SMD [2], which build on the more established Second-Order Sequential Best Rotation (SBR2) [3]. The newer SMD family of algorithms allows para-Hermitian matrices that result from the calculation of a Space-Time covariance matrix to be diagonalised to a much greater degree than the original SBR2 algorithm. The SMD algorithms achieve this by transferring more energy per iteration from the off-diagonal elements of the para-Hermitian matrix to the diagonal of the zero-lag slice. This has been shown to lead to a significant increase in performance of the algorithm in terms of convergence, shown in the figures. In addition to greater diagonalisation performance, the SMD algorithms allow the order of the defined paraunitary matrices to be constrained relative to the original SBR2 algorithm, also shown in the figure on the right. This is important from the perspective of hardware implementation, as the filterbank ultimately defined by the product of unitary matrices can be significantly shorter in length.

The SMD algorithm is more complex to calculate than the SBR2 owing to a full EVD operation being required on each iteration. Therefore, further algorithm developments have been made to address this complexity by employing EVD approximation methods including Cyclic-by-Row rotations, and also new truncation methods to further restrict growth in the length of the paraunitary matrices calculated. With support from LSSC Industrial Partner Mathworks, PEVD algorithms have been made available to the public in the first Matlab toolbox of PEVD algorithms in late 2014, available at: http://pevd-toolbox.eee.strath.ac.uk/ and linked with Mathworks File Exchange. The PEVD algorithms have been applied to broadband Direction of Arrival applications, as shown in the first figure, using methods such as the Polynomial-MUSIC [4].

Figure 2: Remaining off-diagonal energy vs iterations for various iterative PEVD algorithms.



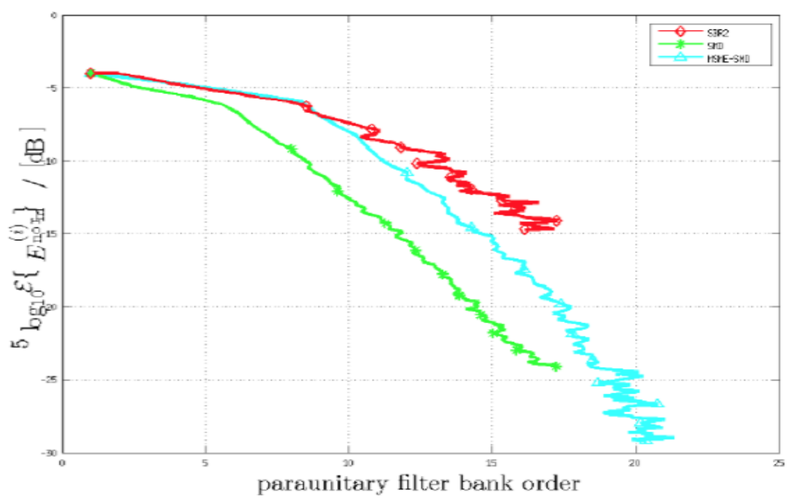


Figure 3: remaining off-diagonal energy vs order of paraunitiary matrix required to achieve this decomposition.

References:

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[2] J. Corr, K. Thompson, S. Weiss, J.G. McWhirter, I.K. Proudler. "Causality-Constrained Multiple Shift Sequential Matrix Diagonalisation for Parahermitian Matrices". In Proceedings of the 22nd European Signal Processing Conference, Nice, France, pp. 1277-1281, Aug./Sept. 2015.

[3] J.G. McWhirter, P. Baxter, T. Cooper, S. Redif and J. Foster. "An EVD Algorithm for Para-Hermitian Polynomial Matrices". In IEEE Transactions on Signal Processing, Volume: 55, Issue: 5, pp. 2158-2169, May 2007.

[4] J. Corr, K. Thompson, S. Weiss, J.G. McWhirter, I.K. Proudler. "Cyclic-By-Row Approximation of Iterative Polynomial EVD algorithms". In Sensor Signal Processing for Defence, Edinburgh, pp. 1-5, Sept. 2014.

[5] S. Weiss, M. Alrmah, S. Lambotharan, J. McWhiter, and M. Kaveh. "Broadband angle of arrival estimation methods in polynomial matrix decomposition framework". In IEEE 5th Int. Workshop Computational Advances in Multisensor Adaptive Processing, St. Martin, pp. 109-112, Dec. 2013.