Robust Multi-Object Sensor Fusion with Unknown Correlations

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***Abstract*—Distribution and decentralisation of fusion operations are key to network centric operations (NCOs) and distributed data fusion algorithms (DDF) have been developed to support them. These algorithms fuse data collected locally with state estimates propagated from other nodes. If the full advantages of NCOs are to be realised, these algorithms should exploit local information only: no single node, for example, should be an oracle which must maintain the entire state of the network. Uhlmann argued that many of these could be overcome if suboptimal solutions were used and proposed a principled suboptimal algorithm known as Covariance Intersection (CI). CI has proved to be a very powerful and general method for fusing data in arbitrary networks and has been used in a range of distributed and other applications where full correlation structures cannot be maintained. However, CI only utilizes the mean and covariance of the estimates and cannot exploit any additional distribution information such as the number of modes. The generalisation of CI to general probability distributions was first proposed by Mahler and independently derived by Hurley. We investigate the generalisation Covariance Intersection for multi-object posteriors by considering specific forms of multi-object posterior and their first-order moment densities, Probability Hypothesis Densities, as a prerequisite study for determining tractable implementations.**

I. Introduction

Current approaches to distributed multi-target multi-source detection (DMMD) incorporate multiple hypothesis tracking (MHT) algorithms with optimal or suboptimal Distributed Data Fusion (DDF) algorithms.

MHT algorithms are an approximation, and require careful tuning to provide acceptable performance. These difficulties are compounded in distributed environments, where individual nodes make track initialisation, track merging and track deletion decisions using local information only. This can cause valid tracks to be pruned, and invalid tracks to be maintained. The main reasons for these difficulties lie in the MHT algorithm itself: it is computationally expensive (data association costs are exponential in the number of measurements) and it is only an approximate (or pseudo-Bayesian) solution. An alternative approach to multi-object tracking is to use a rigorous multi-object multi-sensor detection and classification algorithm. Finite Set Statistics (FISST) [1] has provided the first numerically tractable solution for this problem. FISST uses the Bayesian paradigm to recursively estimate and update a multi-object density function using a set valued measurement received at each time step. There is no need to use complex algorithms for detection, identification, and estimation of objects across multiple sensors. The success of Finite Set Statistics for multi-sensor multi-object tracking has been demonstrated by the Probability Hypothesis Density (PHD) filter algorithms [2], [3] and their Gaussian mixture implementations [4]–[6]. These algorithms replace the exponential complexity of data association techniques with robust, linear-complexity algorithms that are effective in estimating both the correct number of objects and their state vectors in data with high false alarm rates and missed detections.

Distributed data fusion algorithms combine the state estimates that are generated by a number of fusion centres or nodes. However, the estimates from the different nodes are *not* conditionally independent of one another and, if optimal fusion is to occur, common information has to be "cancelled out" [7]. However, in most networks computing this information is prohibitively expensive. An alternative is to use suboptimal fusion techniques. Covariance Intersection (CI) [8] has proved to be a very powerful and general method for fusing data in arbitrary networks and has been used in a range of distributed and other applications where full correlation structures cannot be maintained. However, CI only utilizes the mean and covariance of the estimates and cannot exploit any additional distribution information such as the number of modes. The generalisation of CI to general probability distributions was first proposed by Mahler [9] and the independently derived by Hurley [10]. This generalisation replaces the product form of Bayes Rule with a exponential mixture density (equivalently a weighted geometric mean). Theoretical [11], [12] and practical analysis [13], [14] has demonstrated that this generalisation has a sound theoretical basis. Although Mahler proposed a method for applying FISST in distributed environments [9], his discussion included no proofs of the validity of the proposed method, no implementation strategy of how such a method could be realised, and no demonstration of the performance of the method.

Our aim in this paper, and an accompanying paper on practical implementations [15], is to investigate the multi-object generalisation of Covariance Intersection proposed by Mahler: Firstly, in this paper, by establishing tractable mathematical representations using specific multi-object posteriors and secondly, in the accompanying paper [15], by examining the behaviour of these approaches in challenging multi-sensor scenarios. The paper is structured as follows: In the next section, we describe the foundations of multi-object Finite-Set Statistics. In Section III, we investigate tractable forms of the generalised form of Covariance Intersection for multiple target densities. The paper concludes in Section IV.

II. Finite Set Statistics

In this section we describe the background material on multi-object posteriors and their first-moment densities.

A. Multi-object Density Functions

A multi-object density function $f(X)$ is a real-valued function of a finite set variable $X = \{x_1, \ldots, x_N\}$ [1]. Both the number of points and the spatial locations of random finite set X are random. The set integral over multi-object density function is given by,

$$\int f(X)\delta X := f(\emptyset) + \sum_{n=1}^{\infty} \frac{1}{n!} \int f(\{x_1, \ldots, x_n\}) dx_1 \ldots dx_n, \tag{1}$$

where $f(\{x_1, \ldots, x_n\})$ is defined as the joint distribution scaled by its cardinality,

$$f(\{x_1, \ldots, x_n\}) := n! \cdot p(n) \cdot f(x_1, \ldots, x_n). \tag{2}$$

The $n!$ factorial term accounts for the fact we need to consider all permutations in the joint distribution. The cardinality distribution $p(n)$ is a discrete probability distribution in the number of objects and satisfies $\sum_{n=0}^{\infty} p(n) = 1$.

Evaluation of the set integral in (1) is computationally prohibitive for most engineering applications and hence any multi-object estimation problems that require the computation of an integral, such as the multi-object generalisation of the Bayes filter, will require approximations. One such approximation commonly used in tracking application is the first-order moment statistic, known as the Probability Hypothesis Density (PHD) filter [2]. Moments of the multi-object distribution can be determined via the probability generating functional (p.g.fl.), which we now define.

B. Probability Generating Functionals

Multi-object density functions can be uniquely characterised in terms of their probability generating functional (p.g.fl.). The probability generating functional (p.g.fl.) of a (multi-object) probability distribution $f(X)$ on random finite set X is defined as the expectation value of the symmetric function

$$h^X := \prod_{x \in X} h(x), \tag{3}$$

where $h \leq 1$ is a test function, so that the p.g.fl. is

$$G[h] := E\left(h^X\right) = \int f(X) \cdot h^X \delta X. \tag{4}$$

The set derivative is defined by taking the functional derivatives with respect to each point x in point process $X = \{x_1 \ldots x_m\}$, i.e.

$$\frac{\delta G[h]}{\delta X} := \frac{\delta G[h]}{\delta x_1 \ldots \delta x_m} \tag{5}$$

Moments of multi-object posterior can be obtained by taking the set-derivative of the p.g.fl. G at $h = 1$, i.e.

$$D(X) = \left. \frac{\delta G}{\delta X}[h] \right|_{h=1}. \tag{6}$$

In particular, the first-order moment of multi-object probability density $f(X)$, more commonly known as the Probability Hypothesis Density in the target tracking literature [2], can be found by taking the functional derivative of its p.g.fl. G evaluated at $h = 1$, i.e.

$$D(x) = \left. \frac{\delta}{\delta x} G[h] \right|_{h=1}. \tag{7}$$

The PHD is a function on the single-object state space whose integral in any particular region gives the expected number of objects in that region.

C. Common Multi-Object Distributions

We now consider three multi-object distributions that are commonly used in multi-object tracking applications due to their convenient mathematical representations: The Bernoulli process, used in the IPDA filter [16] and Joint Target Detection and Tracking filters [1], [17], the Poisson process, used in the PHD filter [2], and the independent, identically distributed (i.i.d.) cluster process used in the CPHD filter [3]. The Bernoulli process considers scenarios where there is at most one target, so that the Bayes filter is still tractable. The Poisson and i.i.d. processes are convenient since their moment-densities can be computed in the PHD and CPHD filter updates respectively.

Multi-object Density Example 1 - Bernoulli processes: This multi-object distribution considers the scenario where there is at most one target in the scene. Under this assumption, we compute an exact multi-object Bayes filter since the computational complexity of the set integral is not prohibitively expensive.

Suppose that we have a probability density function f on a random set X, where X contains random variable x with probability p, with x distributed according to probability density $f(x)$, or it is empty with probability $1-p$, i.e.

$$f(X) = \begin{cases} 1-p, & X=\emptyset \\ p\cdot f(x), & X=\{x\} \end{cases} \tag{8}$$

We refer to this process as a Bernoulli process. It can be shown that this density integrates to 1 as follows.

$$\begin{aligned} \int f(X)\delta X :&= f(\emptyset) + \int f(\{x\})dx \\ &= (1-p) + p\int f(x)dx = 1. \end{aligned} \tag{9}$$

The probability generating functional of a Bernoulli process is given by

$$G[h] = \int h^Y f(X)\delta X = f(\emptyset) + \int h(x) f(\{x\})dx \tag{10}$$

The PHD of a Bernoulli process is characterised by the first order statistical moment of the p.g.fl. evaluated at $h=1$,

$$D(x) = \frac{\delta}{\delta x} G[h]\bigg|_{h=1} = f(\{x\}) = p\cdot f(x). \tag{11}$$

Multi-object Density Example 2 - Independent, identically distributed (i.i.d.) cluster processes: Let $p(n)$ be the cardinality distribution of the point process X and let $f(y)$ be a probability density function on state space $\mathcal{X}$. For any random set $Y=\{y_1,\ldots,y_n\}$ with $|Y|=n$, define the multi-object probability distribution

$$f(Y) := n!\cdot p(n)\cdot f(y_1)\ldots f(y_n). \tag{12}$$

The p.g.fl. of an independent identically distributed (i.i.d.) point process is

$$G[h] = \sum_{n=0}^{\infty} p(n)\left(\int h(u)\cdot f(u)du\right)^n. \tag{13}$$

The intensity function, or PHD, of an i.i.d. process is found with

$$D(x) = \frac{\delta}{\delta x} G[h]\bigg|_{h=1} = f(x)\sum_{n=1}^{\infty} n\cdot p(n), \tag{14}$$

which is the product of the single-object density $f(x)$, and the expected number of objects $\sum_{n=1}^{\infty} n\cdot p(n)$.

Multi-object Density Example 3 - Poisson point processes: Suppose that in equation (12), that the cardinality distribution $p(n)$ is given by

$$p(n) = \frac{\exp(-\lambda)\lambda^n}{n!}, \tag{15}$$

and the multi-object distribution is

$$f(Y) := e^{-\lambda}\cdot\lambda^n\cdot\prod_{y_i\in Y} f(y_i). \tag{16}$$

The p.g.fl. of a Poisson point process is given by

$$G[h] = \exp\left(\mu\int h(x)f(x)dx - \mu\right) \tag{17}$$

The intensity function, or PHD, of a Poisson point process is found by taking the functional derivative of (17), evaluated at $h=1$, i.e.

$$D(x) := \frac{\delta}{\delta x} G[h]\bigg|_{h=1} = \mu\cdot f(x), \tag{18}$$

where μ gives the expected number of objects that are distributed according to $f(x)$.

III. Generalised Covariance Intersection for Multi-Object Posteriors

The generalisation of Covariance Intersection was proposed by Mahler specifically to extend FISST to suboptimal distributed environments [9], and this generalisation has proved to be extremely valuable for distributed estimation in the single-target case [11]–[14]. However, despite the fact that these equations were introduced for multi-object posteriors, no authors to date have attempted to analyse or apply these equations.

For robust distributed data fusion for fusion of two multi-object posteriors, $f_0(X|Z_0^k)$ and $f_1(Y|Z_1^k)$, that are conditioned on measurement set sequences, Z_0^k and Z_1^k, from two different sensor suites, into a fused multi-object posterior, $f_\omega(X|Z_0^k,Z_1^k)$, Mahler proposed the following generalisation of Covariance Intersection [9],

$$f_\omega(X|Z_0^k,Z_1^k) = \frac{f_0(X|Z_0^k)^{(1-\omega)} f_1(X|Z_1^k)^\omega}{\int f_0(Y|Z_0^k)^{(1-\omega)} f_1(Y|Z_1^k)^\omega \delta Y}, \tag{19}$$

and an approach for finding the optimal value of parameter $0\le\omega\le1$ in an information-theoretic sense that yields a consistent improvement in performance over either sensor individually. Note that the above integral is not an integral in the conventional sense but rather a *set-integral* that integrates over all joint target-spaces, considering each cardinality (number of targets) [1]. Thus this integral is not practical for more than a few targets and so approximation strategies, such as the PHD filter [2], are required. In the next sections, we derive explicit formulae for specific tractable types of fused multi-object posterior and first-moment densities as a prerequisite for investigating the multi-object fusion rules in practical scenarios.

A. Robust fusion of Bernoulli Posteriors

In this section, we show that when the multi-object generalisation of Covariance Intersection is restricted to the joint target-detection and estimation problem, we can find an explicit expression for the resulting distribution.

Theorem: Let us assume that the posteriors f_0 and f_1 are Bernoulli processes, i.e.

$$f_0(X) = \begin{cases} 1-p_0, & X=\emptyset \\ p_0 f_0(x), & X=\{x\}. \end{cases} \tag{20}$$

$$f_1(X) = \begin{cases} 1 - p_1, & X = \emptyset \\ p_1 f_1(x), & X = \{x\}. \end{cases} \quad (21)$$

Then we can evaluate equation (21) for $X = \emptyset$ and $X = \{x\}$ explicitly. The fused probability of target existence and track density become

$$p_\omega = 1 - \frac{1}{K}(1-p_0)^{(1-\omega)}(1-p_1)^\omega \quad (22)$$

$$f_\omega(x|Z_0^k, Z_1^k) = \frac{1}{p_\omega K} p_0^{(1-\omega)} p_1^\omega f_0(x|Z_0^k)^{(1-\omega)} f_1(x|Z_1^k)^\omega. \quad (23)$$

Proof We first evaluate the denominator in equation (21) for Bernoulli posteriors by expanding the set integral,

$$\begin{aligned} K =& f_0(\emptyset|Z_0^k)^{(1-\omega)} f_1(\emptyset|Z_1^k)^\omega + \\ & \int f_0(\{y\}|Z_0^k)^{(1-\omega)} f_1(\{y\}|Z_1^k)^\omega dy \\ =& (1-p_0)^{(1-\omega)}(1-p_1)^\omega + \\ & p_0^{(1-\omega)} p_1^\omega \int f_0(y|Z_0^k)^{(1-\omega)} f_1(y|Z_1^k)^\omega dy. \end{aligned} \quad (24)$$

Then, for the case with no targets, i.e. $X = \emptyset$, we have

$$\begin{aligned} f_\omega(\emptyset|Z_0^k, Z_1^k) &= \frac{1}{K} f_0(\emptyset|Z_0^k)^{(1-\omega)} f_1(\emptyset|Z_1^k)^\omega \\ &= \frac{1}{K}(1-p_0)^{(1-\omega)}(1-p_1)^\omega, \end{aligned} \quad (25)$$

whereas for the single-target case, i.e. $X = \{x\}$, we have

$$\begin{aligned} f_\omega(\{x\}|Z_0^k, Z_1^k) &= \frac{1}{K} f_0(\{x\}|Z_0^k)^{(1-\omega)} f_1(\{x\}|Z_1^k)^\omega \\ &= \frac{1}{K} p_0^{(1-\omega)} p_1^\omega f_0(x|Z_0^k)^{(1-\omega)} f_1(x|Z_1^k)^\omega. \end{aligned} \quad (26)$$

The resulting existence probablity and track density follow from

$$p_\omega = 1 - f_\omega(\emptyset|Z_0^k, Z_1^k), \quad (27)$$

$$f_\omega(x|Z_0^k, Z_1^k) = \frac{f_\omega(\{x\}|Z_0^k, Z_1^k)}{p_\omega}. \quad (28)$$

B. *Robust fusion of Poisson multi-object intensities*

We now consider Poisson multi-object intensities. The PHD filter makes the assumption that the multi-object process is Poisson in order to get a closed form solution for the PHD update. Here we consider the scenario where the posterior multi-object PHD on each sensor is Poisson before fusion with the generalised Covariance Intersection.

Theorem: Let us assume that the posteriors f_0 and f_1 are Poisson processes, i.e.

$$\begin{aligned} f_0(X) &= \exp(-\mu_0) \prod_{x \in X} \mu_0 \cdot s_0(x), \\ f_1(X) &= \exp(-\mu_1) \prod_{x \in X} \mu_1 \cdot s_1(x), \end{aligned} \quad (29)$$

where μ_0 and μ_1 are the Poisson average number of objects, each distributed according to spatial single-object density $s_0(x)$ and $s_1(x)$ respectively. Then the PHD of the fused posterior becomes

$$D_\omega(x) = \mu_\omega \cdot s_\omega(x), \quad (30)$$

where the expected number of objects and location density are computed with

$$\mu_\omega = \mu_0^{(1-\omega)} \mu_1^w \cdot \int s_0^{(1-\omega)}(y) s_1^w(y) dy, \quad (31)$$

$$s_\omega(x) = \frac{s_0^{(1-\omega)}(x) s_1^w(x)}{\int s_0^{(1-\omega)}(y) s_1^w(y) dy}. \quad (32)$$

Proof Substituting the Poisson posteriors into equation (21), we get

$$\frac{1}{K} \exp\left(-\mu_0(1-w) - \mu_1 w\right) \prod_{x \in X} \left(\mu_0 \mu_1 \cdot s_0^{(1-\omega)} s_1^w\right). \quad (33)$$

where the normalising constant K is equal to

$$\exp\left(-\mu_0(1-w) - \mu_1 w + \mu_0^{(1-\omega)} \mu_1^\omega \int s_0^{(1-\omega)}(y) \cdot s_1^w(y) dy\right) \quad (34)$$

The p.g.fl. of the fused multi-object posterior is

$$G_\omega[h] = \frac{1}{K} \int h^X f_0^{(1-\omega)}(X|Z_0^{(k)}) f_1^\omega(X|Z_1^{(k)}) \delta X \quad (35)$$

$$= \frac{1}{K} \int h^X (\exp(-\mu_0)\mu_0 \cdot s_0^X)^{(1-\omega)} (\exp(-\mu_1)\mu_1 \cdot s_1^X)^\omega \delta X$$

$$= \frac{1}{K} \exp\left(-\mu_0(1-w) - \mu_1 w + \mu_0 \mu_1 \int h(y) s_0^{(1-\omega)}(y) s_1^w(y) dy\right)$$

Taking the PHD of $f_\omega(X|Z_0^k, Z_1^k)$, we get

$$D_\omega(x) = \mu_0^{(1-\omega)} \mu_1^w \cdot s_0^{(1-\omega)}(x) s_1^w(x). \quad (36)$$

This leads to the fused expected number of objects and density as described.

Remark: Note that if we consider the single-target case in equation (31), i.e. if $\mu_0 = \mu_1 = 1$, then $\mu_\omega \neq 1$ in general, and hence the fused intensity is no longer a probability distribution. This could lead to a poor estimate of the estimated number of targets with this approach.

C. *Robust fusion of i.i.d. cluster intensities*

We now relax the assumption of having a Poisson cardinality distribution to allow for arbitrary cardinality distributions with i.i.d. cluster processes. In the CPHD filter, the predicted multi-object distribution was assumed to be an i.i.d. cluster process before computing the CPHD filter update.

Theorem: Let us assume that the posteriors f_0 and f_1 are i.i.d. cluster processes, i.e.

$$f_0(X) = n! \cdot p_0(n) \prod_{x\in X} s_0(x) \tag{37}$$
$$f_1(X) = n! \cdot p_1(n) \prod_{x\in X} s_1(x)$$

Then the fused PHD is

$$D_\omega(x) = s_\omega(x) \cdot \sum_{n=1}^{\infty} n \cdot p_\omega(n), \tag{38}$$

where the updated i.i.d. location density and cardinality distribution are

$$s_\omega(x) = \frac{s_0^{(1-\omega)}(x) s_1^\omega(x)}{\int s_0^{(1-\omega)}(y) s_1^\omega(y) dy} \tag{39}$$

$$p_\omega(n) = \frac{p_0^{(1-\omega)}(n) p_1^w(n) \left(\int s_0^{(1-\omega)}(x') s_1^\omega(x') dx'\right)^n}{\sum_{m=0}^\infty p_0^{(1-\omega)}(m) p_1^{(w)}(m) \left(\int s_0^{(1-\omega)}(y) s_1^\omega(y) dy\right)^m}. \tag{40}$$

Proof Substiting the i.i.d. posteriors into equation (21) gives

$$\frac{1}{K} n! \cdot p_0^{(1-\omega)}(n) p_1^{(w)}(n) \prod_{x\in X} s_0^{(1-\omega)}(x) s_1^{(w)}(x), \tag{41}$$

where the normalising constant K is equal to

$$\sum_{n=0}^\infty \frac{1}{n!} \int f_0^{(1-\omega)}(\{x_1,\dots,x_n\}) f_1^\omega(\{x_1,\dots,x_n\}) dx_1 \dots dx_n \tag{42}$$
$$= \sum_{n=0}^\infty p_0^{(1-\omega)}(n) p_1^{(w)}(n) \left(\int s_0^{(1-\omega)}(x) s_1^\omega(x) dx\right)^n.$$

The PHD is computed with (see [1, p584])

$$\begin{aligned} D_\omega(x) =& \frac{1}{K} \int f_0^{(1-\omega)}(\{x\} \cup X) \cdot f_1^\omega(\{x\} \cup X) \delta X \qquad (43) \\ =& \frac{1}{K} \sum_{n=0}^\infty \frac{1}{n!} (n+1)! p_0^{(1-\omega)}(n+1) p_1^w(n+1) \times \\ & s_0^{(1-\omega)}(x) s_1^\omega(x) \prod_{i=1}^n \int s_0^{(1-\omega)}(x_i) s_1^\omega(x_i) dx_i \\ =& \frac{1}{K} s_0^{(1-\omega)}(x) s_1^\omega(x) \times \\ & \sum_{n=1}^\infty n p_0^{(1-\omega)}(n) p_1^w(n) \left(\int s_0^{(1-\omega)}(x') s_1^\omega(x') dx'\right)^{n-1}. \end{aligned}$$

The resulting cardinality distribution and location density follow.

Remark: Note that if we consider the fusion of two single-object distributions, the estimated number of targets for the fused PHD in equation (38) scales correctly to 1 in this case, which indicates that this method is likely to better estimate the number of targets than the Poisson PHD case.

IV. Conclusions

This paper investigates potentially numerically tractable forms of the generalisation of the Covariance Intersection algorithm to multi-object densities. We have shown that, for several widely-used classes of distributions, exact closed form solutions for the distributions exist. Given that closed forms exist, the next job is to provide efficient implementations. We consider these in the companion paper [15].

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References

[1] R. P. S. Mahler, "Statistical Multisource Multitarget Information Fusion," *Artech House*, 2007.
[2] R. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, No.4, pp. 1152–1178, 2003.
[3] ——, "PHD Filters of Higher Order in Target Number," *IEEE Trans. AES. Vol. 43 No 4*, pp. 1523–1543, 2007.
[4] B. Vo and W. K. Ma, "The Gaussian Mixture Probability Hypothesis Density Filter," *IEEE Transactions on Signal Processing*, vol. 54, no. 11, pp. 4091–4104, 2006.
[5] B. T. Vo, B.-N. Vo, and A. Cantoni, "Analytic Implementations of Probability Hypothesis Density Filters." *IEEE Transactions on Signal Processing, Vol 55 No 7 Part 2*, pp. 3553–3567, 2007.
[6] D. E. Clark and B. Vo., "Convergence Analysis of the Gaussian Mixture PHD Filter." *IEEE Trans. Sig. Proc.*, vol. 55, No. 4, pp. 1204 – 1212, 2007.
[7] C.-Y. Chong, S. Mori, and K.-C. Chang, "Distributed multitarget multisensor tracking," *Multitarget-Multisensor Tracking: Advanced Applications; Y. Bar-Shalom (ed.); Artech House Chapter 8*, 1990.
[8] J. K. Uhlmann, "Dynamic map building and localization for autonomous vehicles," Ph.D. dissertation, University of Oxford, 1995.
[9] R. Mahler, "Optimal/robust distributed data fusion: a unified approach," *SPIE Defense and Security Symposium*, 2000.
[10] M. Hurley, "An information-theoretic justification for covariance intersection and its generalization," in *Proceedings of the 2002 FUSION Conference*, Annapolis, MD, USA, 7–11 July 2002.
[11] S. J. Julier, "Fusion without independence (keynote abstract)," in *Proceedings of the IET Seminar on Tracking and Data Fusion: Algorithms and Applications*, Birmingham, UK, 15–16 April 2008, pp. 1–5.
[12] S. J. Julier, T. Bailey, and J. K. Uhlmann, "Using exponential mixture models for suboptimal distributed data fusion," in *Proceedings of the 2006 IEEE Nonlinear Statistical Signal Processing Workshop (NSSPW'06)*, Cambridge, UK, 13–15 September 2006, pp. 160–163.
[13] S. Julier, "An Empirical Study into the Use of Chernoff Information for Robust, Distributed Fusion of Gaussian Mixture Models," *IEEE Int. Conf. on Information Fusion*, 2006.
[14] L.-L. Ong, T. Bailey, and H. Durrant-Whyte, "Decentralised particle filtering for multiple target tracking in wireless sensor networks," in *Proceedings of the 11th International Conference on Information Fusion*, Cologne, Germany, 30th June – 3rd July 2008, pp. 1–8.
[15] M. Uney, S. Julier, D. Clark, and B. Ristic, "Monte Carlo Realisation of A Distributed Multi-Object Fusion Algorithm," *Sensor Signal Processing for Defence*, 2010.
[16] D. Mušicki and R. Evans and S. Stankovic, "Integrated probabilistic data association," *IEEE Trans. Automatic Control*, vol. 39, no. 6, pp. 1237–1240, 1994.
[17] B. T. Vo, "Random Finite Sets in Multi Object Filtering," *PhD Thesis, The University of Western Australia*, October 2008.